Solution

PRE-BOARD 2023-24

Class 10 - Mathematics

Section A

1. **(a)** equal

   Explanation: If we assume that a and b are equal and consider a = b = k

   Then,

   HCF (a, b)= k

   LCM (a, b) = k

2. **(b)** $x^2 - 3x - 10 = 0$

   Explanation: Sum of the roots = 5 + (-2) = 3, product of roots = 5 $\times$ (-2) = -10.

   $\therefore$ x^2 - (sum of roots) x + product of roots = 0.

   Hence, $x^2 - 3x - 10 = 0$.

3. **(c)** $\tan^2 A$

   Explanation: $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$

   $= \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}$

   $= \left(1+\tan^2 A\right)\left(\frac{\tan^2 A}{\tan^2 A+1}\right) = \tan^2 A$

   Hence, the correct choice is $\tan^2 A$.

4. **(d)** $\frac{1}{9}$

   Explanation: The number of possible outcomes when two dice are thrown is 36.

   Now, the possible outcomes of getting a product of 12 are

   {(2, 6),(3, 4),(4, 3),(6, 2)}, which means the number of favourable outcome is 4.

   Required probability = $\frac{4}{36}$ = $\frac{1}{9}$

5. **(a)** $\frac{5}{7}$

   Explanation: No. of days in a leap year = 366

   No. of Mondays = 52

   Extra days = 366 - 52 $\times$ 7

   = 366 - 364 = 2

   $\therefore$ Remaining days in the week = 7 - 2 = 5

   $\therefore$ Probability of being 52 Mondays in the leap year = $\frac{5}{7}$

6. **(c)** A is true but R is false.

   Explanation: Here, reason is not true.

   $\sqrt{9} = \pm 3$, which is not an irrational number.

   A is true but R is false.

7. **(a)** Both A and R are true and R is the correct explanation of A.

   Explanation: Both are correct. Reason is the correct explanation.

   Assertion,

   $a_n = 7 - 4n$

   $d = a_{n-1} - a_n$

   = 7 - 4(n + 1) - (7 - 4n)

= 7 - 4n - 4 - 7 + 4n = -4

8. **(a)** 3

Explanation: The number of zeroes is 3 as the graph given in the question intersects the x-axis at 3 points.

9.

(d) 10

Explanation: In $\triangle$ADE and $\triangle$ABC

$\angle$D = $\angle$B {Corresponding angle}

$\angle$E = $\angle$C {Corresponding angle}

$\therefore$ $\triangle$ADE and $\triangle$ABC (by A A Similarity)

$\frac{AD}{AB}$ = $\frac{DE}{BC}$

$\frac{2}{5}$ = $\frac{4}{X}$

X = $\frac{5\times4}{2}$ = 10

= 10 cm

10.

(d) (x, y)

Explanation: AB = $\sqrt{(2x-0)^2+(0-2y)^2}$

= $\sqrt{4x^2+4y^2}$ = $2\sqrt{x^2+y^2}$ units

BO = $\sqrt{(0-2x)^2+(0-0)^2}$

= $\sqrt{4x^2}$ = 2x units

AO = $\sqrt{(0-0)^2+(0-2y)^2}$

= $\sqrt{4y^2}$ = $2y$ units

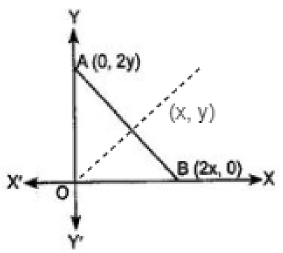
Now, $\mathrm{AB}^2=\mathrm{AO}^2+\mathrm{BO}^2 \Rightarrow \left(2\sqrt{x^2+y^2}\right)^2=(2x)^2+(2y)^2$

$\Rightarrow 4\left(x^2+y^2\right)=4\left(x^2+y^2\right)$

Therefore, triangle AOB is an isosceles right-angled triangle.

Since the coordinate of the point which is equidistant from the three vertices of a right-angled triangle is the coordinates of mid-point of its hypotenuse.

$\therefore$ Mid-point of AB = $\left(\frac{0+2x}{2}, \frac{2y+0}{2}\right)$ = (x,y)



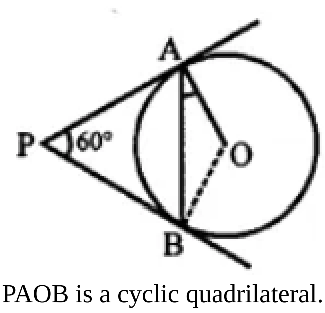
11.

(b) 30°

Explanation: In the given figure, PA and PB are two tangents to the circle with centre O.

$\angle$APB = 60°

To find $\angle$OAB,Join OB.



PAOB is a cyclic quadrilateral.

$\angle$APB + $\angle$AOB = 180°

OA is radius and PA is tangent

OA ⊥ AP ⇒ ∠OAP = 90°
PA = PB (Tangents to the circle)
∠PAB = ∠PBA
But, ∠PAB + ∠PBA = 180° – 60° = 120°
∠PAB = ∠PBA = $\frac{120}{2}$ = 60°
∠OAB = 90° – 60° = 30°

12.
(d) 27
Explanation:

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	18	20	17	7	4
Cumulative Frequency	4	9	27	47	64	71	75

Therefore, the number of athletes who completed the race in less than 125 seconds is 27.

Section B

13. -21
Explanation:
The given polynomial f(x) = $2x^2$ + x + k
If 3 is zero of f(x) then f(3) = 0
i,e. $2(3)^2$ + 3 + k = 0
⇒ 2 × 9 + 3 + k = 0
⇒ 18 + 3 + k = 0
∴ 21 + k = 0
⇒ k = -21
Thus, for k = -21, 3 is a zero of the polynomial.

14. 19
Explanation:
Here it is given an AP
where a = 72 and d = -4
Suppose the n^{th} term = 0
T_n = a + (n - 1)d = 0
So 72 + (n - 1)(-4) = 0
72 - 4n + 4 = 0
-4n = -72 - 4 = -76
$n = \frac{-76}{-4} = 19$
Hence, the 19^{th} term of the given AP is 0.

15. 5
Explanation:
According to question it is given that In $\triangle ABC$,
MN || AB
Therefore by Thale's theroem
$\frac{MC}{AC} = \frac{NC}{BC}$
$\Rightarrow \frac{MC}{AM+MC} = \frac{NC}{BC}$
$= \frac{x}{7.5}$ (when NC = x cm)
$\Rightarrow x = \frac{2\times7.5}{6}$
$= \frac{15}{6}$ = 2.5
⇒ NC = 2.5 cm

Hence, BN = BC - NC
= (7.5 - 2.5) cm
= 5 cm

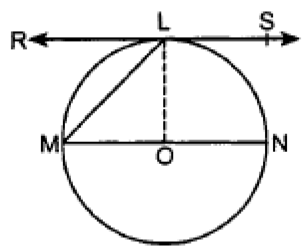
16. 29
Explanation:
Point C(-1, 2) divides internally the line segment A(2, 5) and B(x, y) in the ratio 3 : 4
Then, by section formula
$C=\left(\frac{3\times x+4\times 2}{3+4},\frac{3\times y+4\times 5}{3+4}\right)$
$\Rightarrow(-1,2)=\left(\frac{3x+8}{7},\frac{3y+20}{7}\right)$
$\Rightarrow\frac{3x+8}{7}=-1\text{ and }\frac{3y+20}{7}=2$
⇒ 3x + 8 = -7 and 3y + 20 = 14
⇒ 3x = -15 and 3y = -6
⇒ x = -5 and y = -2
∴ $x^2 + y^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$

17. 28
Explanation:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of students	20	24	40	36	20
		f_0	f_1	f_2	

Modal class = 20 - 30
l = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10
Mode = $l+\frac{(f_1-f_0)}{2f_1-f_0-f_2}\times h$
$=20+\frac{(40-24)}{2(40)-24-36}\times 10$
$=20+\frac{(40-24)}{80-24-36}\times 10$
$=20+\frac{16\times 10}{20}$
$=20+\frac{160}{20}$
= 20 + 8
= 28

18. 60
Explanation:
Given,



Construction: Join OL
OL ⊥ RS.
Also OL = OM
∴ ∠OML = ∠OLM
⇒. ∠OLM = 30^o
⇒ ∠RLM = $90^o - 30^o = 60^o$

Section C

19. Given, $p = a^2b^3$

and $q = a^3b$

$HCF(p,q) = a^2b$

$LCM(p, q) = a^3b^3$

$pq = a^2b^3 \times a^3b = a^5b^4 \;------------ (1)$

$LCM(p,q) \times HCF(p,q) = a^3b^3 \times a^2b = a^5b^4 ----(2)$

$From\ equation\ (1)\ and\ (2)\ We\ get$

$LCM(p,q) \times HCF(p,q) = pq$

20. We have to find the zeroes of the quadratic polynomial $4y^2 – 15$ and verify the relationship between the zeroes and coefficient of polynomial.

Let $f(y) = 4y^2 - 15$

Compare it with the quadratic $ay^2 + by + c$.

Here, coefficient of $y^2 = 4$, coefficient of y = 0 and constant term = - 15.

Now $4y^2 - 15 = (2y)^2 - (\sqrt{15})^2$

$= (2y + \sqrt{15})(2y - \sqrt{15})$

The zeroes of f(y) are given by $f(y) = 0$

$\Rightarrow (2y) + \sqrt{15})(2y - \sqrt{15}) = 0$

$\Rightarrow (2y) + \sqrt{15}) = 0$ or $(2y - \sqrt{15}) = 0$

$\Rightarrow 2y = -\sqrt{15}$ or $2y = \sqrt{15}$

$\Rightarrow y = -\frac{\sqrt{15}}{2}$ or $y = \frac{\sqrt{15}}{2}$

Hence, the zeroes of the given quadratic polynomial are $-\frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2}$

Verification of relationship between zeroes and coefficients

Sum of the zeroes = $-\frac{\sqrt{15}}{2} + \frac{\sqrt{15}}{2} = \frac{-\sqrt{15} + \sqrt{15}}{2} = \frac{0}{2} = 0 = \frac{0}{4}$

$= \frac{coefficient\ of\ y}{coefficient\ of\ y^2}$

Product of zeroes = $-\frac{\sqrt{15}}{2} \times \frac{\sqrt{15}}{2} = -\frac{15}{4} = \frac{constant\ term}{coefficient\ of\ y^2}$.

21. Let S_1 and S_2 be two squares. Let the side of the square S_2 be x cm in length. Then,

the side of square S_1 is (x + 4) cm.

Therefore, area of square $S_1 = (x + 4)^2$ [Because, Area = $(side)^2$]

and, Area of square $S_2 = x^2$

It is given that

Area of square S_1 + Area of square S_2 =400 cm^2

$\Rightarrow (x + 4)^2 + x^2 = 400$

$\Rightarrow (x^2 + 8x + 16) + x^2 = 400$

$\Rightarrow 2x^2 + 8x - 384 = 0$

$\Rightarrow x^2 + 4x -192 = 0$

$\Rightarrow x^2 + 16x - 12x -192 = 0$

$\Rightarrow x(x + 16) - 12(x + 16) = 0$

$\Rightarrow (x + 16) (x - 12) = 0$

$\Rightarrow x = 12$ or, $x = -16$

As the length of the side of a square cannot be negative. Therefore, x = 12.

Therefore, side of square S_1 = x + 4 = 12 + 4 = 16 cm and, Side of square S_2 = 12 cm.Hence the side of square S_1 and S_2 are 16 cm and 12 cm respectively.

22. Let first term be a and common difference be d

Given 5^{th} term = 30

$\Rightarrow a + (5 - 1)d = 30$

$\Rightarrow a + 4d = 30$ .... (i)

and, 12^{th} term = 65

$\Rightarrow$ a + (12 - 1)d = 65

$\Rightarrow$ a + 11d = 65 .... (ii)

Subtracting equation (i) from equation (ii)

a + 11d - a - 4d = 65 - 30

$\Rightarrow$ 7d = 35

$\Rightarrow d = \frac{35}{7} = 5$

Putting value of d in equation (i)

a + 4 $\times$ 5 = 30

$\Rightarrow$ a = 30 - 20 = 10

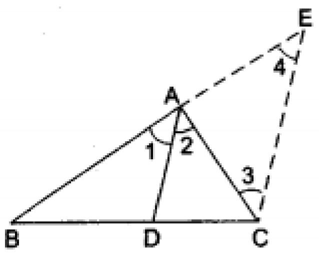
$\therefore$ Sum of first 20 terms $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2 \times 10 + (20-1) \times 5]$

= 10[20 + 95]

= 10 $\times$ 115

= 1150

23. 

It is given that in $\triangle ABC$, AD, the bisector of $\angle A$ meets BC in D.

To Prove $\frac{BD}{DC} = \frac{AB}{AC}$

Construction Draw $CE \| DA$, meeting BA produced at E.

Proof: Since $DA \| CE$, we have

$\angle 2 = \angle 3$ ....(alternative interior angles)

and $\angle 1 = \angle 4$ ..... (corresponding angles)

But, $\angle 1 = \angle 2$ ....(Because AD is bisector of $\angle A$)

$\therefore \quad \angle 3 = \angle 4$

So, AE = AC [since sides opposite to equal sides of a triangle are equal].

Now, in $\triangle BCE$, we have $DA \| CE$.

Therefore by basic proportionality theorem, we have

$\frac{BD}{DC} = \frac{AB}{AE}$

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AC}$ [$\because$ AE = AC]

or $\frac{BD}{DC} = \frac{AB}{AC}$

Hence proved

24. In equilateral $\triangle$ABC , coordinates of points A and B are (2,0) and (5,0) respectively.we have to find the co-ordinates of the other two vertices.

Let co-ordinates of C be (x, y)

Since $AC^2 = BC^2$ (sides of equilateral triangle)

$(x - 2)^2 + (y - 0)^2 = (x - 5)^2 + (y - 0)^2$

or, $x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$

or, 6x = 21

x = $\frac{7}{2}$

And $(x - 2)^2 + (y - 0)^2 = 9$

or, $\left(\frac{7}{2} - 2\right)^2 + y^2 = 9$

or, $\frac{9}{4} + y^2 = 9$

or, $y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$

Hence, C = $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$

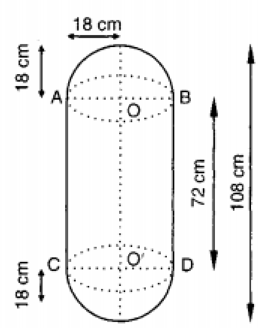
25. Given radius of hemispherical ends = 18 cm

Height of body (h - 2r)= 108 - 2(18) = 108 - 36 = 72 cm

Curved surface area of cylinder = 2πr

Let S be the total surface area of the solid. Then,

S = Curved surface area of the cylinder + Surface areas of hemispherical end



$\Rightarrow \quad S=\left(2\pi rh+2\times 2\pi r^2\right)\text{ cm}^2$

$\Rightarrow \quad S=\left(2\pi rh+4\pi r^2\right)\text{ cm}^2$

$\Rightarrow \quad S=2\pi r(h+2r)\text{cm}^2$

$\Rightarrow \quad S=2\times\frac{22}{7}\times 18\times(72+36)\text{cm}^2$ [$\because$r = 18 cm, h = 72 cm]

$\Rightarrow \quad S=2\times\frac{22}{7}\times 18\times 108\text{cm}^2$ =12219 cm^2

Rate of polishing = 7 paise per sq. cm.

$\therefore$ Cost of polishing = Rs$\left(12219.42\times\frac{7}{100}\right)$=Rs. 885.36

26. We have

Class interval	Frequency (f_i)	Class mark (x_i)	$f_i x_1$
0 - 20	5	10	50
20-40	8	30	240
40-60	x	50	50x
60-80	12	70	840
80-100	7	90	630
100-120	8	110	880
Total	$\Sigma f_i=40+x$		$\Sigma f_i x_i=2640+50x$

Here, $\Sigma f_i x_i=2640+50x$, $\Sigma f_i=40+x$, $\bar{X}=62.8$

$\therefore \quad \text{Mean}(\bar{X})=\frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 62.8=\frac{2640+50x}{40+x}$

$\Rightarrow$ 2512 + 62.8x = 2640 + 50x

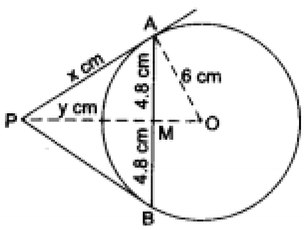
$\Rightarrow$ 62.8x - 50x = 2640 - 2512

$\Rightarrow$ 12.8x = 128

$\therefore \quad x=\frac{128}{12.8}=10$

Hence, the missing frequency is 10

27. AB is a chord of length 9.6 cm of a circle with centre O and radius 6 cm.



The tangents at A and B intersect at P.

CONSTRUCTION : Join OP and OA. Let OP and AB intersect at M.

Let PA = x cm and PM = y cm.

Now, $PA=PB$

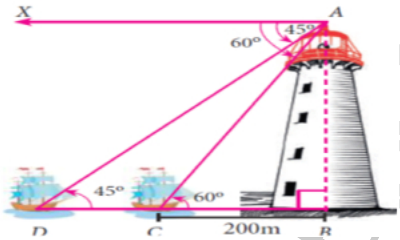
and OP is the bisector of $\angle APB$ [$\because$ two tangents to a circle from an external point are equally inclined to the line segment

joining the centre to that point.
Also, $OP \perp AB$ and OP bisects AB at M [$\because$ OP is the right bisector of AB]
$\therefore$ AM = MB = $\frac{9.6}{2}$cm
$= 4.8cm.$
In right $\triangle AMO$, we have
$OA = 6cm$
and $AM = 4.8cm.$
$\therefore$ OM = $\sqrt{OA^2 - AM^2}$
$= \sqrt{6^2 - 4.8^2}$
$= \sqrt{12.96}$
$= 3.6\text{cm}.$
In right $\triangle PAO$, we have
$AP^2 = PM^2 + AM^2$
$\Rightarrow x^2 = y^2 + (4.8)^2$
$\Rightarrow \quad x^2 = y^2 + 23.04$ ...(i)
In right $\triangle PAO$, we have
$OP^2 = PA^2 + OA^2$ [Note $\angle PAO = 90^\circ$, since AO is the radius at the point of contact]
$\Rightarrow \quad (y + 3.6)^2$
$= x^2 + 6^2$
$\Rightarrow \quad y^2 + 7.2y + 12.96$
$= x^2 + 36$
$\Rightarrow 7.2y = 46.08$ [using (i)]
$\Rightarrow \quad y = 6.4\text{cm}$
and
$x^2 = (6.4)^2 + 23.04$
= 40.96 + 23.04 = 64
$\Rightarrow x = \sqrt{64} = 8$
$\therefore PA = 8cm.$

Section D

28. **Read the text carefully and answer the questions:**

A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60^o with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45^o.



(i) In $\triangle$ ABC
$\tan 60^o = \frac{AB}{BC}$
$\sqrt{3} = \frac{AB}{200}$
AB = $200\sqrt{3}$
Now, In $\triangle$ ABD
$\tan 45^o = \frac{AB}{BD}$
1 = $\frac{200\sqrt{3}}{BD}$
BD = $200\sqrt{3}$
$\therefore$ CD = BD - BC
= $200\sqrt{3}$ - 200
= 200 ($\sqrt{3}$ - 1)
= 200 $\times$ (1.732 - 1)
= 200 $\times$ 0.732

= 146.4 m

speed = $\frac{distance}{time}$

= $\frac{146.4}{10}$

= 14.64 m/s

Now,

speed = 14.64 $\times$ $\frac{18}{5}$ km/hr

= 52.7

$\approx$ 53 km/hr

(ii) In $\triangle$ ABD

tan 45^o = $\frac{AB}{BD}$

1 = $\frac{200\sqrt{3}}{BD}$

BD = $200\sqrt{3}$ m

$\therefore$ CD = $200\sqrt{3}$ - 200

= 200 ($\sqrt{3}$ - 1)

= 200 (1.732 - 1)

= 200 $\times$ 0.732

= 146.4

$\approx$ 147 m

$\therefore$ boat is at a distance of 147 m from its actual position.

(iii) In $\triangle$ ABC

tan 60^o = $\frac{AB}{BC}$

$\sqrt{3}$ = $\frac{AB}{200}$

AB = $200\sqrt{3}$ m

Hence, height of tower = $200\sqrt{3}$ m

(iv) As boat moves away from the tower angle of depression decreases.

Section E

29. The given system of equations is

0.4x + 0.3y = 1.7 ....(i)

0.7x - 0.2y = 0.8 ....(ii)

Multiplying both sides of (i) and (ii), by 10, get

4x + 3y = 17 ....(iii)

7x - 2y = 8 ....(iv)

From (iv), we get

7x = 8 + 2y

$\Rightarrow x = \frac{8+2y}{7}$

Substituting $x = \frac{8+2y}{7}$ in (iii), we get

$4\left(\frac{8+2y}{7}\right) + 3y = 17$

$\Rightarrow \frac{32+8y}{7} + 3y = 17$

$\Rightarrow \frac{32+8y+21y}{7} = 17$

$\Rightarrow 32 + 29y = 17 \times 7$

$\Rightarrow 29y = 119 - 32$

$\Rightarrow 29y = 87$

$\Rightarrow y = \frac{87}{29} = 3$

Putting y = 3 in $x = \frac{8+2y}{7}$, we get

$x = \frac{8+2\times3}{7}$

$= \frac{8+6}{7}$

$= \frac{14}{7}$

$x = 2$

Hence, the solution of the given system of equations is x = 2 and y = 3.

30. We have,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90 - \cos 45 + \cos 60^\circ)$ …… (1)

Now,

$\sin 90^\circ = \cos 0^\circ = 1, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

So by substituting above values in equation (1)

We get, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$

Now, LCM of both the product terms in the above expression is $2\sqrt{2}$

Therefore we get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} + \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right) \times \left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} - \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right)$

$= \left(\frac{2\sqrt{2}}{2\sqrt{2}} + \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right)$

$= \left(\frac{2\sqrt{2}+2+\sqrt{2}}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}-2+\sqrt{2}}{2\sqrt{2}}\right)$

Now by rearranging terms in the numerator of above expression

We get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(\frac{2\sqrt{2}+\sqrt{2}+2}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}+\sqrt{2}-2}{2\sqrt{2}}\right)$

$= \frac{(2\sqrt{2}+\sqrt{2}+2)\times(2\sqrt{2}+\sqrt{2}-2)}{(2\sqrt{2})\times(2\sqrt{2})}$

Now, by applying formula $[(a+b)(a-b) = a^2 - b^2]$ in the numerator of the above expression we get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \frac{(2\sqrt{2}+\sqrt{2})^2 - 2^2}{2\times 2\times\sqrt{2}\times\sqrt{2}}$

$= \frac{(2\sqrt{2}+\sqrt{2})^2 - 2^2}{4\times 2}$ …… (2)

Now, we know that $(a+b)^2 = a^2 + 2ab + b^2$

Therefore, $(2\sqrt{2}+\sqrt{2})^2 = (2\sqrt{2})^2 + 2(2\sqrt{2})(\sqrt{2}) + (\sqrt{2})^2$

Now, by substituting the above value of $(2\sqrt{2}+\sqrt{2})^2$ in equation (2)

We get, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45 + \cos 60^\circ)$

$= \frac{\left[(2\sqrt{2})^2 + 2(2\sqrt{2})(\sqrt{2}) + (\sqrt{2})^2\right] - 2^2}{4\times 2}$

$= \frac{[8+8+2]-4}{8}$

$= \frac{18-4}{8}$

$= \frac{14}{8}$

Now $\frac{14}{8}$ gets reduced to $\frac{7}{4}$

Therefore,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \frac{7}{4}$

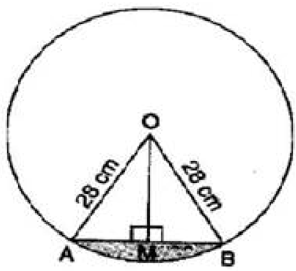
Hence, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ) = \frac{7}{4}$

31. r = 28 cm and θ = $\frac{360}{6} = 60^\circ$

Area of minor sector = $\frac{\theta}{360}\pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$

= 410.67 cm^2

For, Area of $\triangle$AOB,



Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]
OM = OM [Common]
$\therefore \triangle$OMA $\cong$ OMB [RHS congruency]
$\therefore$ AM = BM [By CPCT]
$\Rightarrow$ AM = BM = $\frac{1}{2}$AB and $\angle$AOM = $\angle$BOM = $\frac{1}{2}$AOB = $\frac{1}{2} \times 60^o = 30^o$
In right angled triangle OMA, cos30^o = $\frac{OM}{OA}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$
$\Rightarrow$ OM = $14\sqrt{3}$ cm
Also, sin30° = $\frac{AM}{OA}$
$\Rightarrow \frac{1}{2} = \frac{AM}{28}$
$\Rightarrow$ AM = 14 cm
$\Rightarrow$ 2 AM = 2 $\times$ 14 = 28 cm
$\Rightarrow$ AB = 28 cm
$\therefore$ Area of $\triangle$AOB = $\frac{1}{2} \times$ AB $\times$ OM = $\frac{1}{2} \times 28 \times 14\sqrt{3}$ = 196 $\sqrt{3}$ = 196 $\times$ 1.7 = 333.2 cm^2
$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle$AOB
= 410.67 - 333.2 = 77.47 cm^2
$\therefore$ Area of one design = 77.47 cm^2
$\therefore$ Area of six designs = 77.47 $\times$ 6 = 464.82 cm^2
Cost of making designs = 464.82 $\times$ 0.35 = Rs. 162.68

32. Height of cone (h) = 10 cm
Radius of cone and hemisphere (r) = 7 cm
Slant height of cone (l) = $\sqrt{h^2+r^2}$
l = $\sqrt{10^2+7^2}$ = $\sqrt{100+49}$ = $\sqrt{149}$
l = 12.2cm
Volume of toy = volume of cone + volume of hemisphere
Volume of toy = $\pi r^2 h + \frac{2}{3}\pi r^3$
Volume = $\pi r^2\left(h+\frac{2}{3}r\right) = \frac{22}{7} \times 49 \times \left(10+\frac{2}{3} \times 7\right)$
Volume = $22 \times 7 \times \left(10+\frac{14}{3}\right)$ = $\frac{22\times7\times44}{3}$
Volume = 2258.66 cm^3
Volume of toy = 2258.66cm^3
Now,
Surface area of toy = CSA of cone + CSA of hemisphere
Surface area = $\pi rl + 2\pi r^2$
Surface area = $\pi r(l+2r) = \frac{22}{7} \times 7$ (12.2 + 14)
Surface area= 22 $\times$ 26.2
Surface area = 576.4 cm^2
Surface area of coloured sheet required = 576.4cm^2

33. Total number of cards in one deck of cards = 52
$\therefore$ Total number of outcomes n = 52
   i. Let E_1= Event of getting a king of red color. So number of outcomes favourable to E_1 and m = 2 [$\because$ there are two kings of red color in a deck]
   So, P(E_1) = $\frac{m}{n} = \frac{2}{52} = \frac{1}{26}$
   ii. Let E_2 = Event of getting the queen of the diamond
   $\therefore$ Numbers of outcomes favourable to E_2 = 1 [$\because$ there is only one queen of diamond in a deck]
   Hence, P(E_2)= $\frac{m}{n}$ = $\frac{1}{52}$
   iii. Let E_3 = Event of getting an ace.
   $\therefore$ Number of outcomes favourable to E_3 = 4
   Hence, P(E_3) = $\frac{4}{52}$ = $\frac{1}{13}$