

Solution

PRE-BOARD

Class 10 - Mathematics

Section A

1. (c) 1679
Explanation: The dividend is equal to Divisor \times Quotient + Remainder
Number(dividend) = $D \times Q + R$
 \therefore the number (Dividend) = $61 \times 27 + 32$
= $1647 + 32$
= 1679
2. (b) 0
Explanation: There is no zero as the graph does not intersect the x-axis at any point.
3. (d) 0
Explanation: The number of solutions of two linear equations representing parallel lines is 0 because two linear equations representing parallel lines has no solution and they are inconsistent.
4. (c) real and unequal
Explanation: When $D > 0$, the roots of the given quadratic equation are real and unequal.
5. (a) 47
Explanation: Here, $a = 2$, $d = 7 - 2 = 5$ and $n = 10$
 $\therefore a_n = a + (n - 1)d$
 $\Rightarrow a_{10} = 2 + (10 - 1) \times 5$
= $2 + 9 \times 5$
 $\Rightarrow a_{10} = 2 + 45 = 47$
6. (d) $\sqrt{52}$
Explanation: Let us take $(3, -2)$ and $(-3, 2)$ as (x_1, y_1) and (x_2, y_2)
Using distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $d = \sqrt{(-3 - 3)^2 + (2 - (-2))^2}$
 $d = \sqrt{(-6)^2 + (2 + 2)^2}$
 $d = \sqrt{36 + (4)^2}$
 $d = \sqrt{36 + 16}$
 $d = \sqrt{52}$
7. (d) $(-4, 2)$
Explanation: $(x, y) = \left\{ \frac{(-6+(-2))}{2}, \frac{(8+(-4))}{2} \right\}$
= $\left(\frac{-8}{2}, \frac{4}{2} \right)$
= $(-4, 2)$
8. (a) 5 cm.
Explanation: In triangles APB and CPD,
 $\angle APB = \angle CPD$ [Vertically opposite angles] $\angle BAP = \angle ACD$ [Alternate angles as $AB \parallel CD$]
 $\therefore \triangle APB \sim \triangle CPD$ [AA similarity]
 $\therefore \frac{AB}{CD} = \frac{CP}{AP}$

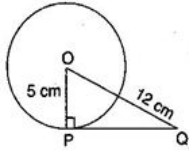
$$\Rightarrow \frac{4}{6} = \frac{AP}{7.5}$$

$$\Rightarrow AP = \frac{7.5 \times 4}{6} = 5 \text{ cm}$$

9.

(d) $\sqrt{119}$ cm.

Explanation:



$\angle OPQ = 90^\circ$ [Angle between tangent and radius through the point of contact]

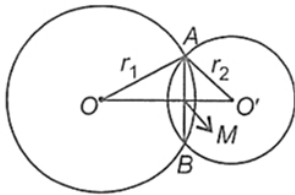
$$\therefore OQ^2 = OP^2 + PQ^2 \Rightarrow (12)^2 = (5)^2 + PQ^2$$

$$\Rightarrow PQ^2 = 144 - 25 \Rightarrow PQ^2 = 119$$

$$\Rightarrow PQ = \sqrt{119}$$

10. (a) $(8 + 2\sqrt{7})$ cm

Explanation: Since, M is the mid-point of AB.



$$\therefore AM = 6 \text{ cm}$$

$$AO(r_1) = 10 \text{ cm}, AO'(r_2) = 8 \text{ cm}$$

AB is perpendicular to OO' , then

$$\text{In } \triangle AOM, \text{ using Pythagoras theorem, } 100 = 36 + OM^2$$

$$\Rightarrow OM = 8 \text{ cm};$$

$$\text{In } \triangle AMO', 64 = 36 + O'M^2$$

$$\Rightarrow \sqrt{28} = O'M \Rightarrow 2\sqrt{7} = O'M$$

$$\therefore OO' = (2\sqrt{7} + 8) \text{ cm}$$

11.

(b) $\sin 60^\circ$

$$\text{Explanation: } \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

$$= \sin 60^\circ$$

12. (a) 1

Explanation: We have, $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$

$$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{1 - \sin^2 \theta}{\sin \theta} \times \frac{1 - \cos^2 \theta}{\cos \theta} \times \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

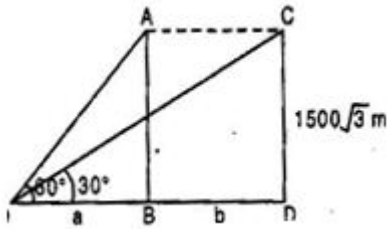
$$= \frac{\cos^2 \theta}{\sin \theta} \times \frac{\sin^2 \theta}{\cos \theta} \times \frac{1}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = 1$$

13.

(c) 720 km/hr

Explanation:



Let A be the initial position and C be the final position of the aeroplane. And $\angle APB = 60^\circ$, $\angle CPD = 30^\circ$ and $AB = CD = 1500\sqrt{3}$ m

Let $PB = a$ meters and $BD = b$ meters Then $PD = PB + BD = (a + b)$ m

In right angled triangle APB, we have

$$\tan 60^\circ = \frac{AB}{PB}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{a}$$

$$\Rightarrow a = 1500 \text{ m}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{(a+b)}$$

$$\Rightarrow a + b = 4500$$

$$\Rightarrow 1500 + b = 4500$$

$$\Rightarrow b = 3000 \text{ m}$$

\therefore Distance covered by the aeroplane in 15 seconds = $AC = BD = 3000$ m

Now, Speed of aeroplane = $\frac{3000}{15} = 200$ m/s = 720 km/hr

The speed of the plane is 720 km/hr.

14.

(b) $\frac{10\pi}{13}$

Explanation: $\frac{10\pi}{13}$

15.

(b) 72°

Explanation: It is given that area of the sector = 69.3 cm^2 and Radius = 10.5 cm

$$\text{Now, Area of the sector} = \frac{\pi r^2 \theta}{360}$$

$$\Rightarrow \frac{\pi \times (10.5)^2 \times \theta}{360} = 69.3$$

$$\Rightarrow \theta = \frac{69.3 \times 360 \times 7}{10.5 \times 10.5 \times 22} = 72^\circ$$

Therefore, Central angle of the sector = 72°

16.

(b) $\frac{3}{10}$

Explanation: Total numbers are $\sum x_i = 10$

x	f
3	1
5	2
7	3
9	4

$$\text{Average} = \frac{3 \times 1 + 5 \times 2 + 7 \times 3 + 9 \times 4}{10}$$

$$= \frac{3 + 10 + 21 + 36}{10} = \frac{70}{10} = 7$$

$$\therefore m = 3$$

$$\therefore \text{Probability of average number} = \frac{3}{10}$$

17. (a) $\frac{1}{26}$

Explanation: black kings = club king + spade king = 2

Number of possible outcomes = 2
 Number of Total outcomes = 52
 \therefore Required Probability = $\frac{2}{52} = \frac{1}{26}$

18.

(d) 30 – 40

Explanation: According to the question,

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Freq	3	9	15	30	18	5

Here Maximum frequency is 30.

Therefore, the modal class is 30 – 40.

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

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Section B

21. Consider $\sqrt{p} + \sqrt{q}$ is rational and can be represented as $\sqrt{p} + \sqrt{q} = a$

$$\Rightarrow (\sqrt{p}) = a - \sqrt{q}$$

$$\Rightarrow (\sqrt{p})^2 = (a - \sqrt{q})^2 \text{ (squaring both sides)}$$

$$\Rightarrow p = a^2 + (\sqrt{q})^2 - 2a\sqrt{q}$$

$$\Rightarrow p = a^2 + q - 2a\sqrt{q}$$

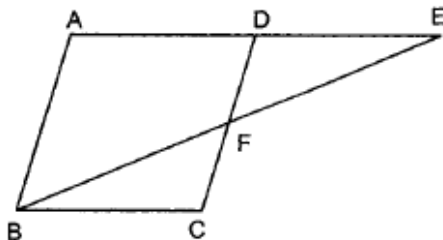
$$\Rightarrow 2a\sqrt{q} = a^2 + q - p$$

$$\Rightarrow \sqrt{q} = \frac{a^2 + q - p}{2a}$$

As q is prime so \sqrt{q} is not rational but $\frac{a^2 + q - p}{2a}$ is rational because a, p, q are non-zero integers which contradicts our consideration.

Hence, $\sqrt{p} + \sqrt{q}$ is irrational where p and q are primes.

22. In Δ 's ABE and CFB, we have



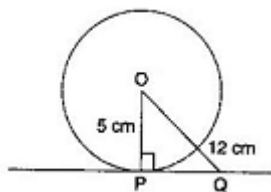
$\angle AEB = \angle CBF$ [Alternate angles]

$\angle A = \angle C$ [Opposite angles of a parallelogram]

Thus, by AA-criterion of similarity, we have,

$\triangle ABE \sim \triangle CFB$

23. \therefore PQ is the tangent and OP is the radius through the point of contact.



$\therefore \angle OPQ = 90^\circ$

[The tangent at any point of a circle is perpendicular to the radius through the point of contact]

By Pythagoras theorem in right $\triangle OPQ$,

$$OQ^2 = OP^2 + PQ^2$$

$$\Rightarrow (12)^2 = (5)^2 + PQ^2$$

$$\Rightarrow 144 = 25 + PQ^2$$

$$\Rightarrow PQ^2 = 144 - 25$$

$$\Rightarrow PQ^2 = 119$$

$$\Rightarrow PQ = \sqrt{119} \text{ cm}$$

Hence, the length PQ is $\sqrt{119}$ cm.

24. To verify: $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

If $A = 60^\circ$ and $B = 30^\circ$, then

To verify: $\tan 90^\circ = \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$

Consider R.H.S. = $\frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}}$

$$= \frac{3+1}{0} = 4/0 = \infty$$

$$= \tan 90^\circ = \text{L.H.S.}$$

$$\therefore \text{R.H.S.} = \text{L.H.S.}$$

Hence, verified.

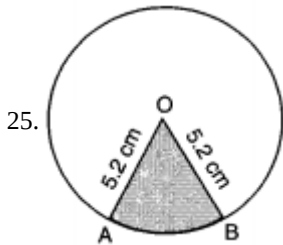
OR

$$\text{LHS} = (\sec \theta + \tan \theta)(1 - \sin \theta)$$

$$= \left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right) (1 - \sin \theta)$$

$$= \left(\frac{1 + \sin \theta}{\cos \theta} \right) (1 - \sin \theta) = \frac{(1 - \sin^2 \theta)}{\cos \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta} = \cos \theta = \text{RHS}$$



Let OAB be the given sector.

It is given that Perimeter of sector OAB = 16.4 cm

$$\Rightarrow OA + OB + \text{arc AB} = 16.4 \text{ cm}$$

$$\Rightarrow 5.2 + 5.2 + \text{arc AB} = 16.4$$

$$\Rightarrow \text{arc AB} = 6 \text{ cm}$$

$$\Rightarrow l = 6 \text{ cm}$$

$$\therefore \text{Area of sector OAB} = \frac{1}{2}lr = \frac{1}{2} \times 6 \times 5.2 \text{ cm}^2 = 15.6 \text{ cm}^2$$

OR

Angle described by the minute hand in 60 minutes = 360°

$$\text{Angle described by the minute hand in 20 minutes} = \left(\frac{360}{60} \times 20 \right)$$

$$= 120^\circ$$

Required area swept by the minute hand in 20 minutes

= Area of the sector (with $r = 15 \text{ cm}$ and $\theta = 120^\circ$)

$$= \left(\frac{\pi r^2 \theta}{360^\circ} \right) \text{ cm}^2$$

$$= \left(3.14 \times 15 \times 15 \times \frac{120^\circ}{360^\circ} \right)$$

$$= 235.5 \text{ cm}^2$$

Section C

26. The greatest number of cartons is the HCF of 144 and 90

Now the prime factorization of 144 and 90 are

$$144 = 16 \times 9 = 2^4 \times 3^2$$

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

\therefore The greatest number of cartons each stack would have = 18.

27. $P(x) = 2x^2 - 4x + 5$

Here, $a = 2$, $b = -4$, $c = 5$

Let zeroes be α, β

$$\text{Sum of zeroes } \alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$$

$$\text{Product of zeroes } \alpha \times \beta = \frac{c}{a} = \frac{5}{2}$$

$$\text{i. } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (2)^2 - 2\left(\frac{5}{2}\right)$$

$$= 4 - 5$$

$$\Rightarrow \alpha^2 + \beta^2 = -1$$

$$\text{ii. } (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= (2)^2 - 4\left(\frac{5}{2}\right)$$

$$= 4 - 2(5)$$

$$= 4 - 10$$

$$= -6$$

$$(\alpha - \beta)^2 = -6$$

28. The given equations are

$$3x + 2y = 14$$

$$y = \frac{14-3x}{2}$$

when $x = 0$, then $y = 7$

when $x = 4$, then $y = 1$

when $x = 2$, then $y = 4$

Table for $3x + 2y = 14$

x	0	4	2
y	7	1	4

And $x - 4y = -7$

$$x = 4y - 7$$

When $y = 0$, then $x = -7$

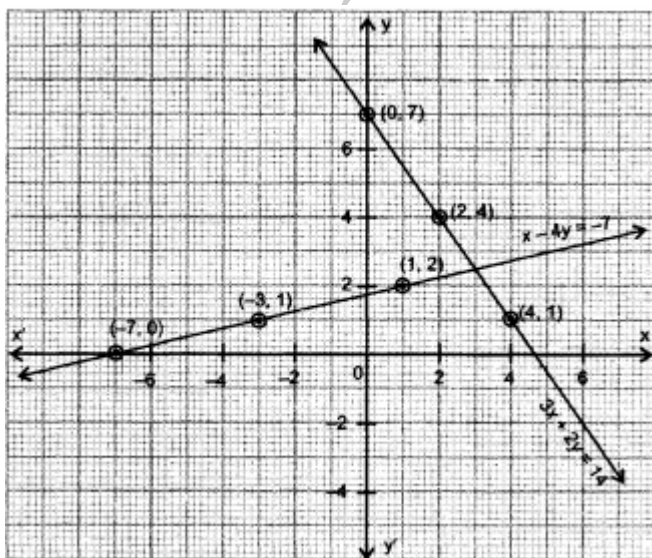
when $y = 1$, then $x = -3$

when $y = 2$, then $x = 1$

Table for $x - 4y = -7$

x	-7	-3	1
y	0	1	2

Represent $3x + 2y = 14$ and $x - 4y = -7$ on graph paper.



The solution of given lines is $x = 3, y = 2.5$.

OR

Let the present ages of A and B be 'a' and 'b' respectively.

As per given conditions, we get the following equations

$$\Rightarrow a + 10 = 2(b + 10)$$

$$\text{and } a - 5 = 3(b - 5)$$

$$\Rightarrow a = 2b + 10 \dots\dots(1)$$

$$\text{and } a = 3b - 10 \dots\dots(2)$$

Subtracting equation (1) from (2), gives

$$b - 20 = 0$$

$$\Rightarrow b = 20$$

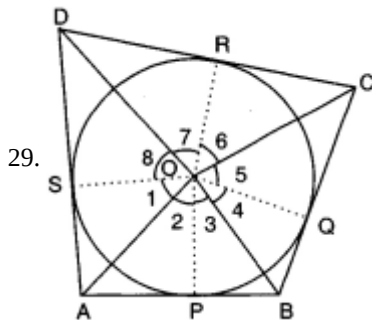
Using this value in (i), gives

$$a = 2b + 10$$

$$\Rightarrow a = 2 \times 20 + 10$$

$$\Rightarrow a = 50$$

Therefore, a = 50, and b = 20.



29.

GIVEN: A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE $\angle AOB + \angle COD = 180^\circ$ and $\angle AOD + \angle BOC = 180^\circ$

CONSTRUCTION Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6 \text{ and } \angle 7 = \angle 8$$

$$\text{Now, } \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ \left[\begin{array}{l} \text{Sum of all the angles} \\ \text{subtended at a point is } 360^\circ \end{array} \right]$$

$$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ \text{ and } 2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^\circ$$

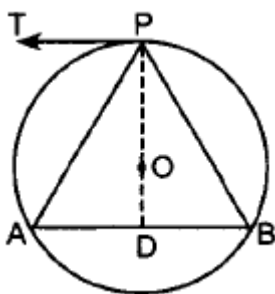
$$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ \text{ and } (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$

$$\Rightarrow \angle AOB + \angle COD = 180^\circ \left[\begin{array}{l} \because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD \\ \angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC \end{array} \right]$$

$$\text{and, } \angle AOD + \angle BOC = 180^\circ$$

OR

Given,



Construction: Join PO and produce it to D.

Proof: Here, $OP \perp TP$

$$\angle OPT = 90^\circ$$

Also, $TP \parallel AB$

$$\therefore \angle TPD + \angle ADP = 180^\circ$$

$$\Rightarrow \angle ADP = 90^\circ$$

OD bisects AB [Perpendicular from the centre bisects the chord]

In $\triangle ADP$ and $\triangle BDP$

$$AD = BD$$

$$\angle ADP = \angle BDP \text{ [Each } 90^\circ]$$

$$PD = PD$$

$$\therefore \triangle ADP \cong \triangle BDP \text{ [SAS]}$$

$$\angle PAB = \angle PBA \text{ [C.P.C.T.]}$$

$\therefore \triangle PAB$ is isosceles triangle.

30. We have,

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

Squaring both sides, we get

$$\Rightarrow \left(\tan \theta + \frac{1}{\tan \theta} \right)^2 = 2^2$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 \times \tan \theta \times \frac{1}{\tan \theta} = 4$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 = 4$$

$$\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} = 2$$

Alternate method, We have

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

$$\Rightarrow \tan^2 \theta + 1 = 2 \tan \theta$$

$$\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0$$

$$\Rightarrow (\tan \theta - 1)^2 = 0$$

$$\Rightarrow \tan \theta = 1$$

$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = 1 + 1 = 2$$

31.	Life time (in hrs)	0 - 100	100 - 200	200 - 300	300- 400	400 - 500
	No. of bulls	5	12	14	6	13

As we may observe that maximum class frequency is 14 belonging to class interval 200 - 300.

So,

Lower limit (l) of modal class = 200

Frequency (f_1) of modal class = 14

Class size (h) = 100

Frequency (f_0) of class preceding the modal class = 12

Frequency (f_2) of class succeeding the modal class = 6

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 200 + \left(\frac{14 - 12}{2 \times 14 - 12 - 6} \right) \times 100$$

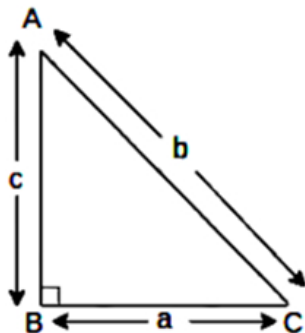
$$= 200 + \frac{200}{10}$$

$$= 200 + 20 = 220$$

Hence modal life = 220 hours.

Section D

32. Let $\triangle ABC$ be the right angle triangle, right angled at B, as shown in figure.



Also, let $AB = c$ cm, $BC = a$ cm and $AC = b$ cm

Then, according to the given information, we have

$$b = 6 + 2a \dots (i) \text{ (Let } a \text{ be the shortest side)}$$

$$\text{and } c = 3a - 6$$

$$\text{We know that, } b^2 = c^2 + a^2$$

$$\Rightarrow (6 + 2a)^2 = (3a - 6)^2 + a^2 \dots [\text{Using (i) and (ii)}]$$

$$\Rightarrow 36 + 4a^2 + 24a = 9a^2 + 36 - 36a + a^2$$

$$\Rightarrow 60a = 6a^2$$

$$\Rightarrow 6a = 60 \dots [\because a \text{ cannot be zero}]$$

$$\Rightarrow a = 10 \text{ cm}$$

Now, from equation (i),

$$b = 6 + 2 \times 10 = 26$$

and from equation (ii),

$$c = 3 \times 10 - 6 = 24$$

Thus, the dimensions of the triangle are 10 cm, 24 cm and 26 cm.

OR

Let two numbers of x and y

According to the question,

$$x + y = 45$$

$$\Rightarrow y = 45 - x \dots (i)$$

$$\text{And } (x - 5)(y - 5) = 124$$

$$(x - 5)[(45 - x) - 5] = 124 \text{ [From equation (i)]}$$

$$(x - 5)(40 - x) = 124$$

$$40x - 200 - x^2 + 5x = 124$$

$$x^2 - 45x + 324 = 0$$

$$x^2 - 36x - 9x + 324 = 0$$

$$x(x - 36) - 9(x - 36) = 0$$

$$(x - 36)(x - 9) = 0$$

$$x - 36 = 0 \text{ or } x - 9 = 0$$

$$x = 36 \text{ or } x = 9$$

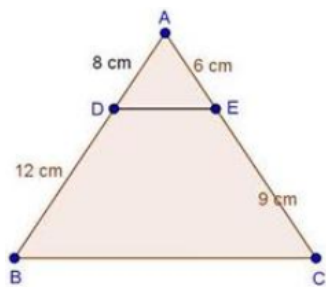
Now from equation (i),

$$\text{When } x = 36, \text{ then } y = 45 - 36 = 9$$

$$\text{When } x = 9, \text{ then } y = 45 - 9 = 36$$

Hence, the numbers are 9 and 36

33. We have,



$$\frac{AD}{DB} = \frac{8}{4} = \frac{2}{1}$$

$$\text{And, } \frac{AE}{EC} = \frac{6}{3} = \frac{2}{1}$$

$$\text{Since, } \frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, according to the converse of basic proportionality theorem, we have

$DE \parallel BC$

In $\triangle ADE$ and $\triangle ABC$

$$\angle A = \angle A \text{ [Common]}$$

$$\angle ADE = \angle B \text{ [Corresponding angles]}$$

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

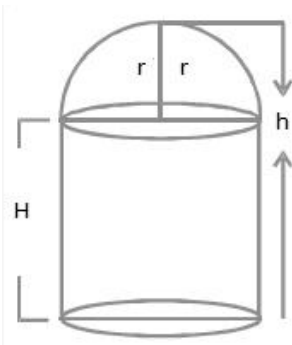
$$\therefore \frac{AD}{AB} = \frac{DE}{BC} \text{ [Corresponding parts of similar } \triangle \text{ are proportional]}$$

$$\Rightarrow \frac{8}{12} = \frac{DE}{BC}$$

$$\Rightarrow \frac{2}{3} = \frac{DE}{BC}$$

$$\Rightarrow BC = \frac{3}{2} DE$$

34.



Let the radius of the hemispherical dome be r and the total height of the building be h .

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height

$$2r = \frac{2}{3}h$$

$$\Rightarrow r = \frac{h}{3}$$

Let H be the height of the cylindrical position.

$$\Rightarrow H = h - r = h - \frac{h}{3} = \frac{2h}{3}$$

Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$$\Rightarrow 67 \frac{1}{21} = \frac{2}{3}\pi r^3 + \pi r^2 H$$

$$\Rightarrow \frac{1408}{21} = \pi r^2 \left(\frac{2}{3}r + H \right)$$

$$\Rightarrow \frac{1408}{21} = \frac{22}{7} \times \left(\frac{h}{3} \right)^2 \left(\frac{2}{3} \times \frac{h}{3} + \frac{2h}{3} \right)$$

$$\Rightarrow \frac{1408 \times 7}{22 \times 21} = \frac{h^2}{9} \times \left(\frac{2h}{9} + \frac{2h}{3} \right)$$

$$\Rightarrow \frac{64}{3} = \frac{h^2}{9} \times \left(\frac{8h}{9} \right)$$

$$\Rightarrow \frac{64 \times 9 \times 9}{3 \times 8} = h^3$$

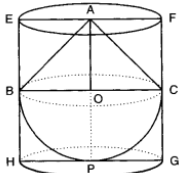
$$\Rightarrow h^3 = 8 \times 27$$

$$\Rightarrow h = 6$$

Thus, the height of the building is 6 m.

OR

Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere. Let EFGH be the right circular cylinder circumscribing the given toy.



We have,

Given radius of cone, cylinder and hemisphere (r) = $\frac{4}{2} = 2$ cm

Height of cone (l) = 2 cm

Height of cylinder (h) = 4 cm

Now, Volume of the right circular cylinder = $\pi r^2 h = \pi \times 2^2 \times 4 \text{ cm}^3 = 16\pi \text{ cm}^3$

Volume of the solid toy = $\left\{ \frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2 \right\} \text{ cm}^3 = 8\pi \text{ cm}^3$

\therefore Required space = Volume of the right circular cylinder - Volume of the toy
 $= 16\pi \text{ cm}^3 - 8\pi \text{ cm}^3 = 8\pi \text{ cm}^3$.

Hence, the right circular cylinder covers $8\pi \text{ cm}^3$ more space than the solid toy.

So, remaining volume of cylinder when toy is inserted in it = $8\pi \text{ cm}^3$

35. Calculation of Median:

Class	Frequency(f_i)	Cumulative frequency
800 - 820	7	7
820 - 840	14	21
840 - 860	19	40

860 - 880	25	65
880 - 900	20	85
900 - 920	10	95
920 - 940	5	100
	$N = \sum f_i = 100$	

Now, $N = 100 \Rightarrow \frac{N}{2} = 50$

Therefore, the cumulative frequency just greater than 50 is 65.

Therefore, the median class = 860 - 880.

$\therefore l = 860, h = 20, f = 25, c. f. = 40$

$$\text{Median, } M = l + \left\{ h \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$$

$$= 860 + \left\{ 20 \times \frac{(50-40)}{25} \right\}$$

$$= 860 + 20 \times \frac{10}{25}$$

$$= 860 + 8 = 868$$

Hence, the median wages is Rs 868.

Section E

36. Read the text carefully and answer the questions:

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$$\frac{5}{1 \text{ coin}}, \frac{10}{2 \text{ coins}}, \frac{15}{3 \text{ coins}}, \frac{20}{4 \text{ coins}}, \frac{25}{5 \text{ coins}}, \dots \text{ to } \frac{n \text{ days}}{n \text{ coins}}$$

We can have at most 190 coins

i.e., $1 + 2 + 3 + 4 + 5 + \dots$ to n term = 190

$$\Rightarrow \frac{n}{2} [2 \times 1 + (n - 1)1] = 190$$

$$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$$

$$\Rightarrow (n + 20)(n - 19) = 0 \Rightarrow (n + 20)(n - 19) = 0$$

$$\Rightarrow n = -20 \text{ or } n = 19 \Rightarrow n = -20 \text{ or } n = 19$$

But number of coins cannot be negative

$\therefore n = 19$ (rejecting $n = -20$)

So, number of days = 19

(ii) Total money she saved = $5 + 10 + 15 + 20 + \dots = 5 + 10 + 15 + 20 + \dots$ upto 19 terms

$$= \frac{19}{2} [2 \times 5 + (19 - 1)5]$$

$$= \frac{19}{2} [100] = \frac{1900}{2} = 950$$

and total money she shaved = ₹950

OR

Number of coins in piggy bank on 15th day

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{15} = \frac{15}{2} [2 \times 5 + (15 - 1) \times 5]$$

$$\Rightarrow S_{15} = \frac{15}{2} [2 + 14]$$

$$\Rightarrow S_{15} = 120$$

So, there are 120 coins on 15th day.

(iii) Money saved in 10 days

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2} [2 \times 5 + (10 - 1) \times 5]$$

$$\Rightarrow S_{10} = 5[10 + 45]$$

$$\Rightarrow S_{10} = 275$$

Money saved in 10 days = ₹275

37. Read the text carefully and answer the questions:

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electrician's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit



- (i) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, grocery store is at (2, 3) and food cart is at (6, -8)

$$\therefore \text{Required distance} = \sqrt{(6-2)^2 + (-8-3)^2}$$

$$= \sqrt{4^2 + 11^2} = \sqrt{16 + 121} = \sqrt{137} \text{ cm}$$

- (ii) Consider the house is at origin (0, 0), then coordinates of the grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Required distance

$$= \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$$

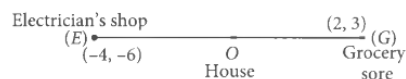
OR

Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, (0, 0) is the mid-point of (-6, 8) and (6, -8), therefore both bus stand and food cart are at equal distances from the house. Hence, required ratio is 1 : 1.

- (iii) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Let O divides EG in the ratio k : 1, then



$$O = \frac{2k-4}{k+1}$$

$$\Rightarrow 2k = 4$$

$$\Rightarrow k = 2$$

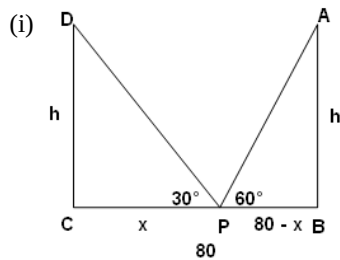
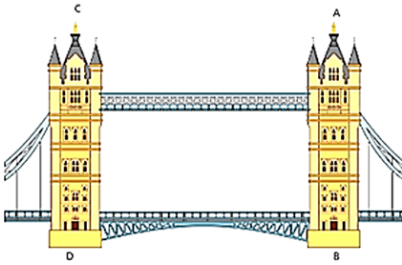
Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE i.e., 1 : 2.

38. Read the text carefully and answer the questions:

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping. In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point

between the towers on the road, the angles of elevation of the top of the towers was 60° and 30° respectively.



Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore $BP = (80 - x)$.

Also, $\angle APB = 60^\circ$ and $\angle DPC = 30^\circ$

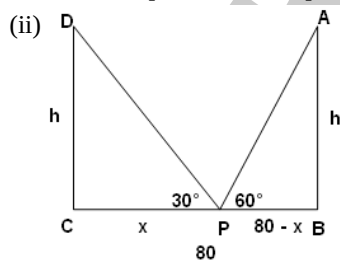
In right angled triangle DCP,

$$\begin{aligned} \tan 30^\circ &= \frac{CD}{CP} \\ \Rightarrow \frac{h}{x} &= \frac{1}{\sqrt{3}} \\ \Rightarrow h &= \frac{x}{\sqrt{3}} \dots\dots(i) \end{aligned}$$

In right angled triangle ABP,

$$\begin{aligned} \tan 60^\circ &= \frac{AB}{BP} \\ \Rightarrow \frac{h}{80-x} &= \sqrt{3} \\ \Rightarrow h &= \sqrt{3}(80 - x) \\ \Rightarrow \frac{x}{\sqrt{3}} &= \sqrt{3}(80 - x) \\ \Rightarrow x &= 3(80 - x) \\ \Rightarrow x &= 240 - 3x \\ \Rightarrow x + 3x &= 240 \\ \Rightarrow 4x &= 240 \\ \Rightarrow x &= 60 \end{aligned}$$

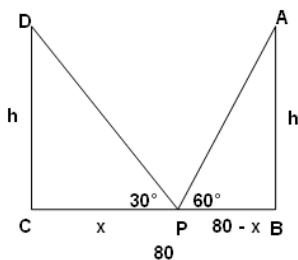
Thus, the position of the point P is 60 m from C.



$$\text{Height of the tower, } h = \frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

The height of each tower is $20\sqrt{3}$ m.

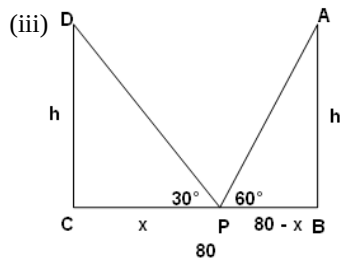
OR



The distance between Neeta and top of tower CD.

In $\triangle CDP$

$$\begin{aligned}\sin 30^\circ &= \frac{CD}{PD} \\ \Rightarrow PD &= \frac{CD}{\sin 30^\circ} \\ \Rightarrow PD &= \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3} \\ \Rightarrow PD &= 40\sqrt{3}\end{aligned}$$



The distance between Neeta and top of tower AB.

In $\triangle ABP$

$$\begin{aligned}\sin 60^\circ &= \frac{AB}{AP} \\ \Rightarrow AP &= \frac{AB}{\sin 60^\circ} \\ \Rightarrow AP &= \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}} \\ \Rightarrow AP &= 40 \text{ m}\end{aligned}$$

RAO BALRAM
SCHOOL